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DEFORMATION OF LAMINATED COMPOSITE PLATES RESTING ON ELASTIC FOUNDATION

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ABSTRACT

In these paper, laminated composite plates resting on elastic foundation is analysed. In these Winkler and Pasternak model is used. The displacement of the plates subjected to sinusoidal loading condition is evaluated. The governing equation is obtained by using principle of virtual work. The plates is analysed for simply supported condition and the results obtain is compared with different shear deformation theories like CPT, FSDT, Reddy's Higher order shear deformation theory.

Keywords: ***elastic foundation, simply supported, sinusoidal, higher order shear deformation theory***

I. INTRODUCTION

Composite materials are widely being used in fields of aerospace and automotive structures because of weight sensitive applications and their high specific strength and stiffness.A lamina or ply is a typical sheet of composite material. A laminate is a collection of laminae stacked to achieve the desired stiffness and thickness. The sequence of various orientations of a fiber-reinforced composite layer in a laminate is termed the lamination scheme or stacking sequence. The layers are usually bonded together with the same matrix material as that in a lamina.The larnination scheme and material properties of individual lamina provide an added flexibility to designers to tailor the stiffness and strength of the laminate to match the structural stiffness and strength requirements. Fiber reinforced composite materials consists of fibers of significant strength and stiffness embedded in a matrix with distinct boundaries between them. Both fibers and matrix maintain their physical and chemical identities, yet their combination performs a functionwhich cannot be done by each constituent acting singly.

With the increasing use of composite materials, the need for advanced methods of analysis became obvious. In case of composite materials, transverse stresses and strains strongly influence the bending behavior. In particular, the transverse shear stress effects are more pronounced. In these theory we are analyzing cross ply orthotropic laminated composite plate on elastic foundation using higher order shear deformation theory and finding out deflection and stresses and strains across the thickness of plates which will give the behavior of the plate under mechanical loading.

Akavci [1] presented the first order shear deformation theory for symmetrically laminated composite plates on elastic foundation. In the classical theory of plates (CPT), it is assumed that plane sections initially normal to the mid surface before deformation remain plane and normal to that surface after deformation. This is the result of neglecting transverse shear strains. However, non-negligible shear deformations occur in thick and moderately thick plates and the theory gives inaccurate results for laminated plates. So, it is obvious that transverse shear deformations have to be taken into account in the analysis. One of the well-known plate theories is the Reissner and Mindlin model which is a first order shear deformation theory (FSDT) and takes the displacement field as linear variations of mid plane displacements.

Pandya and Kant [2] presented finite element analysis of laminated composite plates using a higher-order displacement model. A C^0 continuous displacement finite element formulation of a higher-order theory for flexure of thick arbitrary laminated composite plates under transverse loads is presented. The displacement model accounts for non-linear and constant variation of in-plane and transverse displacement respectively through the plate thickness.

Setoodeh[3] presented a closed form solution for bending and free vibration analyses of simply supported rectangular laminated composite plates is presented. The static and free vibration behavior of symmetric and antisymmetric laminates is investigated using a refined first-order shear deformation theory. The Winkler–Pasternak two-parameter model is employed to express the interaction between the laminated plates and the elastic foundation. The Hamilton’s principle is used to derive the governing equations of motion. The accuracy and efficiency of the theory are verified by comparing the developed results with those obtained using different laminate theories. The effects of the elastic foundation parameters, orthotropy ratio and width-to-thickness ratio on the bending deflection and fundamental frequency of laminates are investigated.

Ghughal[4] used trigonometric shear deformation theory (TSDT) taking into account transverse shear deformation effect as well as transverse normal strain effect is presented. The inplane displacement field uses sinusoidal function in terms of thickness coordinate to include the shear deformation effect. The cosine function in thickness coordinates is used in transverse displacement to include the effect of transverse normal strain. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. The results of displacements and stresses for static flexure of simply supported symmetric and anti-symmetric cross-ply laminated square plates subjected to parabolic load and line load are obtained. The results obtained by present theory are compared with those of classical, first-order and higher-order plate theories.

Sayyad[5] presented cylindrical bending of orthotropic plates is presented using nth-order plate theory. Classical plate theory and parabolic shear deformation theory of Reddy can be considered as special cases of present theory. The theory accounts for realistic variation of the transverse shear stress through the thickness of plate and satisfy the traction free conditions at top and bottom surfaces of the plate. The number of unknown variables in the present theory is same as that of first order shear deformation theory. The theory is variationally consistent. The use of shear correction factors which are problem dependent and are normally associated with first order shear deformation theory is avoided in the present theory. The governing equations and associated boundary conditions are derived by the principle of virtual work. Navier solution technique is employed for the simply supported plates. The program has been developed in FORTRAN. The displacement and stresses of a simply supported plate infinitely long in y-direction under sinusoidally distributed load are calculated to demonstrate the accuracy and efficiency of the present theory.

II. MATHEMATICAL FORMULATION

2.1 Laminated Plate under Consideration

Consider an orthotropic plate of length ‘*a*’ and overall thickness ‘*h*’ as shown in Fig. 1. It is assumed that the plate is of an infinite extent in the *y* direction while it is simply supported at its edges $x = 0$ and $x =$ a. ‘*U*’ is the displacement in *x*-direction while ‘*W*’ is the displacement in *z*-direction. A plate is made up of linearly elastic and orthotropic material. A plate is thin, moderately thick or thick and obeys hook’s law of plane strain problem. A load is applied at the top surface of the plate i.e. $z = -h/2$.

2.2 Displacement Field

The following plate geometry and coordinate system with plate resting on elastic foundation is considered for analytical solutions

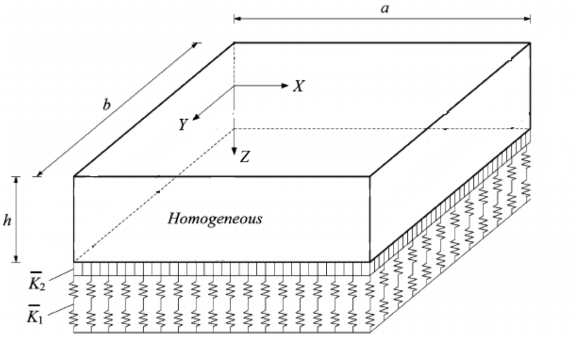


Fig.1 Plate geometry and coordinate system

The displacement field of the laminated composite plates resting on elastic foundation for the plate under consideration is given as below:

$$U(x,y,z) = u_0\left(x,y\right) - z\frac{\partial w}{\partial x} + f(z)\phi(x,y)$$

$$V(x,y,z) = v_0(x,y) - z\frac{\partial w}{\partial y} + f(z)\psi(x,y)$$

$$W(x,y,z) = w_0(x,y) + g(z)\xi(x,y)$$

Where U, V, W are the displacements of the plate and u_0, v_0, w_0 are the displacement of mid-plane in X, Y, Z directions respectively ϕ(x,y), ψ(x,y) and ξ(x,y) are the shear rotations. The function f(z) is chosen based on the shearing stress distribution across the thickness of the plate

2.3 Strain-Displacement Relationship

For the small plate deformation, the six strain components (ε_x, ε_y ,ε_z, γ_{xy}, γ_{zx}, γ_{yz}) and three displacement components (U,V,W) are related according to the well-known linear kinematic relations.

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + f(z)\frac{\partial \psi}{\partial y}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = g'(z)\xi$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + f(z)\left[\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right]$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = g(z)\left[\phi + \frac{\partial \xi}{\partial x}\right]$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = g(z)\left[\psi + \frac{\partial \xi}{\partial y}\right]$$

2.4 Stress Strain Relationship

The stress strain relationship of laminated composite orthotropic plate is given as

$$\begin{Bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{YZ} \\ \tau_{XZ} \\ \tau_{XY} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{YZ} \\ \gamma_{XZ} \\ \gamma_{XY} \end{Bmatrix}$$

$$\begin{aligned} Q_{11} &= (1.0-(\mu_{23}\mu_{32}))/(E_2E_3\Delta) \\ Q_{12} &= (\mu_{21}+(\mu_{31}\mu_{23}))/(E_2E_3\Delta) \\ Q_{13} &= (\mu_{31}+(\mu_{21}\mu_{32}))/(E_2E_3\Delta) \\ Q_{22} &= (1.0-(\mu_{13}\mu_{31}))/(E_1E_3\Delta) \\ Q_{23} &= (\mu_{32}+(\mu_{12}\mu_{31}))/(E_1E_3\Delta) \\ Q_{33} &= (1.0-(\mu_{12}\mu_{21}))/(E_1E_2\Delta) \end{aligned}$$

$$\begin{aligned} Q_{44} &= G_{23} \\ Q_{55} &= G_{13} \\ Q_{66} &= G_{12} \\ \Delta &= 1-\mu_{12}\mu_{21}-\mu_{23}\mu_{32}-\mu_{31}\mu_{13}-2\mu_{21}\mu_{32}\mu_{13} \end{aligned}$$

WhereE_1, E_2, E_3 are the elastic moduli, μ_{12},μ_{21},μ_{23},μ_{32},μ_{31},μ_{13} are Poisson's ratios and G_{12}, G_{23}, G_{13} are the shear moduli of the material.

2.5 Governing equations and boundary conditions:

Principle of virtual work

$$\int_0^a\int_0^b q(x,y)\partial w = \int_0^a\int_0^b\int_{-h/2}^{h/2}\begin{bmatrix} (\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \\ \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz} + k_0 w + k_1\nabla^2 w) \end{bmatrix}$$

The governing equation and boundary condition can be obtained solving above equation by integration byparts and collecting all coefficients the following governing equation are obtained

$$\partial u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\partial v_0 : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\partial w_0 : \frac{\partial^2 M_x^c}{\partial x^2} + 2\frac{\partial^2 M_{xy}^c}{\partial x \partial y} + \frac{\partial^2 M_y^c}{\partial y^2} + q + k_0 w + k_1 \nabla^2 w = 0$$

$$\partial \phi : \frac{\partial M_x^s}{\partial x} + \frac{\partial M_{xy}^s}{\partial x} - Q_x = 0$$

$$\partial \psi : \frac{\partial M_y^s}{\partial y} + \frac{\partial M_{xy}^s}{\partial x} - Q_y = 0$$

$$\partial \xi : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - Q_z = 0$$

2.6 Navier’s Solution:

To prove the efficient and validity of presented theory, bending analysis of simply supported plate resting on wrinkle elastic foundation is considered. The plate is subjected to transverse load q_0(x,y) on the top surface of the plate (i.e. z=-h/2). Six different types of static loading conditions are considered for the detail numerical study.The Navier’s solution technique is employed to determine numerical solution for simply supported plate. The following are the boundary conditions of the simply supported plate.

at edges x = 0, x = a:

at edges y = 0, y = b:

According to Navier solution technique transverse load can be expressed in double trigonometric series.

$$q(x, y) = q_0$$
$$q_0 = q_{mn} \sin \alpha x \sin \beta y$$
$$\alpha = m\pi / a$$
$$\beta = n\pi / b$$

The following is the solution form assume for unknowns variables in displacement fields which satisfies simply supported boundary conditions exactly:

$$u_0 = u_{mn} \cos \alpha x \sin \beta y$$
$$v_0 = v_{mn} \sin \alpha x \cos \beta y$$
$$w_0 = w_{mn} \sin \alpha x \sin \beta y$$
$$\phi = \phi_{mn} \cos \alpha x \sin \beta y$$
$$\psi = \psi_{mn} \sin \alpha x \cos \beta y$$
$$\xi = \xi_{mn} \sin \alpha x \sin \beta y$$
$$q = q_{mn} \sin \alpha x \sin \beta y$$

Where m, n are positive integers. Substituting values of load q(x,y) from equation and putting it in equation we get the following matrix form.

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix}_2 \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \\ \phi_{mn} \\ \psi_{mn} \\ \xi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The elements of stiffness matrix are:

$$\begin{aligned}
K_{11} &= -\left(-A_{11}\alpha^2 + A_{66}\beta^2\right) \\
K_{12} &= -\left(A_{12}\alpha\beta + A_{66}\alpha\beta\right) \\
K_{13} &= \left(B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2\right) \\
K_{14} &= -\left(C_{11}\alpha^2 + C_{66}\beta^2\right) \\
K_{15} &= -\left(C_{12}\alpha\beta + C_{66}\alpha\beta\right) \\
K_{16} &= D_{13}\alpha \\
K_{22} &= -\left(A_{66}\alpha^2 - A_{22}\beta^2\right) \\
K_{23} &= (B_{12}\alpha^2\beta + B_{22}\beta^3 + 2B_{66}\alpha^2\beta) \\
K_{24} &= -\left(C_{12}\alpha\beta + C_{66}\alpha\beta\right) \\
K_{25} &= -\left(C_{66}\alpha^2 + C_{22}\beta^2\right) \\
K_{26} &= D_{23}\beta \\
K_{33} &= -(F_{11}\alpha^4 + 2F_{12}\alpha^2\beta^2 + F_{22}\beta^4 + 4F_{66}\alpha^2\beta^2) \\
&\quad + k_0 + k_1(\alpha^2 + \beta^2) \\
K_{34} &= \left(H_{11}\alpha^3 + H_{12}\alpha\beta^2 + 2H_{66}\alpha\beta^2\right) \\
K_{35} &= \left(H_{22}\beta 3 + 2H_{66}\alpha^2\beta + H_{12}\alpha^2\beta\right) \\
K_{36} &= -\left(I_{13}\alpha^2 + I_{23}\beta^2\right) \\
K_{44} &= -\left(J_{11}\alpha^2 + J_{66}\beta^2 + M_{55}\right) \\
K_{45} &= -\left(J_{12}\alpha\beta + J_{66}\alpha\beta\right)
\end{aligned}$$

$$\begin{aligned}
K_{46} &= \left(K_{13}\alpha - M_{55}\alpha\right) \\
K_{55} &= -\left(J_{22}\beta_2 + J_{66}\alpha^2 + M_{44}\right) \\
K_{56} &= \left(K_{23}\beta - M_{44}\beta\right) \\
K_{66} &= -\left(M_{55}\alpha^2 + M_{44}\beta^2 + S_{33}\right)
\end{aligned}$$

As the matrix is symmetric,

$$\begin{aligned}
&K12 = K21 \quad K13 = K31 \\
&K14 = K41 \quad K15 = K51 \\
&K16 = K61 \quad K23 = K32 \\
&K24 = K42 \quad K25 = K52 \\
&K26 = K62 \quad K34 = K43 \\
&K35 = K53 \quad K36 = K63 \\
&K45 = K54 \quad K46 = K64 \\
&K56 = K65
\end{aligned}$$

The above matrix gives the values of unknowns. Having these values one can then calculate all the displacement and stress components within the plate. Using equations. Transverse shear stresses can be obtained either by constitutive relations or by integrating equilibrium equations of theory of elasticity. The equilibrium equations of theory of elasticity are given below

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial x}=0$$
$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yz}}{\partial z}=0$$
$$\frac{\partial \sigma_z}{\partial z}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{xz}}{\partial x}=0$$

III. NUMERICAL RESULTS AND DISCUSSIONS:

The numerical results are obtained for laminated composite plates subjected to sinusoidal loading condition. The following are the material properties of the plate.

$$E_1=25\ \ \mathrm{E}_2=1\ \ \mathrm{E}_3=1$$
$$\mathrm{G}_{12}=0.5\ \ \mathrm{G}_{13}=0.5\ \mathrm{G}_{23}=0.2$$
$$\mu_{12}=0.25\ \mu_{31}=0.01\ \mu_{23}=0.25$$
$$\mu_{21}=0.01\ \mu_{13}=0.25\ \ \mu_{32}=0.25$$

where 'E' is the Young's modulus and G is the shear modulus in GPa and μ is the Poison's ratio. The transverse deflection (w) are presented in the following normalized form for the purpose of presenting the results in this paper.

$$\overline{\mathrm{w}}(\frac{a}{2},\frac{b}{2},0)=\frac{100\mathrm{E}^2\mathrm{w}}{q_0 h S^4}$$

$$(\overline{\sigma_x},\overline{\sigma_y})=\frac{(\sigma_x,\sigma_y)}{q_0 S^2}$$

$$\overline{\tau_{xy}}(\frac{a}{2},\frac{b}{2},\pm\frac{h}{2})=\frac{\tau_{xy}}{q_0 S^2}$$

$$\overline{\tau_{zx}}(\frac{a}{2},0,0)=\frac{\tau_{zx}}{q_0 S}$$

$$\overline{\tau_{yz}}(0,\frac{b}{2},0)=\frac{\tau_{yz}}{q_0 S}$$

$$K_0=\frac{k_0 b^4}{E_2 h^3}$$

$$K_1=\frac{k_1 b^2}{E_2 h^3}$$

Here S is the aspect ratio (a/h) of the plate. Since exact three dimensional elasticity solution for the bending of laminated composite plate and isotropic plates resting on Winkler elastic foundation is not available in the literature, present results are compared and discussed with the corresponding results of classical plate theory

(CPT), first order shear deformation theory (FSDT), higher order shear deformation theory of Reddy (HSDT). The numerical results are obtained for different values of foundation modulus (K = 3) and aspect ratios (S =4) for isotropic and (K_0=100, K_1=10) and with above same aspect ratio for 0/90 and 0/90/0 cross ply laminated composite plates.

IV. FIGURES AND TABLES

2.1 Simply supported plate subjected to sinusoidal load:

The normalized transverse displacement and isotropic and laminated square plate subjected to sinusoidally distributed load are shown in Table 1. The examination of this table reveals that, the results of displacement obtained by present theory are in good agreement with those of Reddy's theory. The CPT underestimates the value of transverse displacement for aspect ratios 4 and 10.

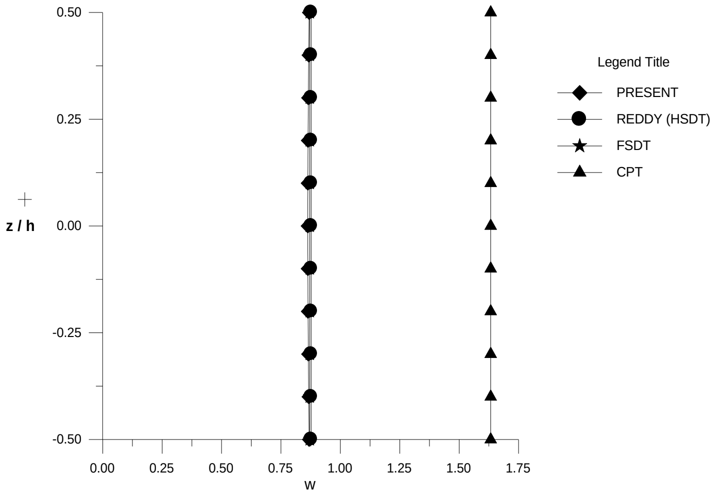
Table 1 Comparison of non-dimensional deflection in laminated composite (0:90) square plate subjected to sinusoidally distributed load

a/h	Model	W
4	Present	0.8684
	Reddy	0.8769
	FSDT	0.8718
	CPT	1.0636
10	Present	1.2089
	Reddy	1.2161
	FSDT	1.2083
	CPT	1.0636
100	Present	1.0643
	Reddy	1.0650
	FSDT	1.0651
	CPT	1.0636
a/h	Model	W
4	Present	0.8622
	Reddy	0.8618
	FSDT	0.7826
	CPT	0.4312
10	Present	0.7154
	Reddy	0.7125
	FSDT	0.6306
	CPT	0.4312
100	Present	0.4342
	Reddy	0.4342

	FSDT	0.4333
	CPT	0.4312

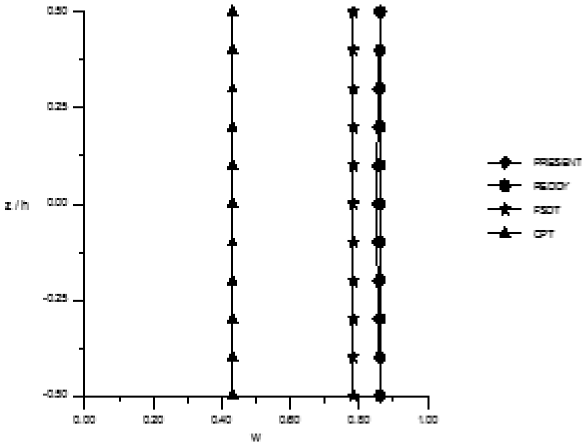
4.2 Graphical results:

The above results is expressed in the graphical form for aspect ratio (S=4) is as follows which gives the clear comparison of the results with other theories:



For 0:90 laminated plate

Graph 2:



V. CONCLUSION

In this paper, the theory is evaluated for analysis of square laminated composite plates resting on Winkler elastic foundation and subjected to sinusoidal loading. The theory is variationally consistent and obviates the need of shear correction factor.The present results are compared with those generated using other theories. From the numerical results and discussionfollowing conclusions are drawn.

The results of displacement predicted by present theory are in excellent agreement with thoseof Reddy’s theory.

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