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NORMAL SHOCK WAVE DIFFRACTION FOR MONOATOMIC GASES

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ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a small bend for γ=1.4, γ being the ratio of specific heats. Srivastava extended the work of Lighthill to general value of r and obtained several results for γ=5/3. In the present paper, pressure distribution over the diffracted shock for γ=5/3 has been obtained.

Keywords: Diffraction, Normal Shock, Monoatomic Gases, Pressure Distribution

I. INTRODUCTION

Lighthill (1949) investigated the diffraction of normal shock wave past a small bend. In this paper the same problem has been treated but the equations have been derived for general value of γ. After the equations have been obtained, the numerical computation has been worked out for the pressure distribution over the diffracted shock wave γ=5/3 (γ being the ratio of specific heats). Earlier Srivastava (1963) have predicted the pressure distribution over the wall as also the curvature of the diffracted shock. The Mach number of the shock wave $M=\acute{\upsilon}/a_o$ ($\acute{\upsilon}$ is velocity of the shock and a_o is the sound speed ahead of the shock wave) has been assumed to be 1.36.

Recently Srivastava (2011) has obtained vorticity distribution over diffracted shock for monoatomic gases. Reference may be made to the book by Srivastava (1994) for detailed reading.

Mathematical Formulation:

Let there be a normal shock of any strength moving into monoatomic gases after meeting a small bend of angle δ gets diffracted.

Let the velocity, pressure, density and sound speed ahead of the shock wave be o, p_o, ρ_o, a_o and behind the shock be q_1, p_1, ρ_1, a_1 before diffraction. Then applying the principle of conservation of mass, momentum and energy across the shock, we obtain:

$$\rho_1(U-q_1) = \rho_o U \qquad ......(1)$$

$$\rho_o U q_1 = p_1 - p_o \qquad ......(2)$$

$$p_1 q_1 = \rho_o U [q_1^2/2 + 1/(\gamma-1)(p_1/\rho_1 - p_0/\rho_o)] \qquad ......(3)$$

From equations (1), (2) and (3) we obtain

$$q_1 = (2U/(\gamma+1))(1-a_o^2/U^2) \qquad ......(4)$$

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$\rho_1 = (\rho_o(\gamma+1))/((\gamma-1)+ 2(a_o^2/ U^2))$ …...(5)

$p_1 = \rho_o/(\gamma+1)[2 U^2-(a_o^2(\gamma -1))/ \gamma]$ …...(6)

where $a_o=(\gamma p_o/ \rho_o)$ is the velocity of sound ahead of shock wave, $M= U / a_o$ is the Mach number of the shock wave.

For $\gamma =5/3$, we obtain from (4), (5) and (6)

$q_1= 3/4 U (1- a_o^2/U^2)$ ……….(7)

$\rho_1= 4 \rho_o / (1 + 3(a_o^2/ U^2))$ ……….(8)

$p_1= 3/4 \rho_o (U^2- (a_o^2/5))$ ………..(9)

$M= q_1/a_1 = q_1/(\sqrt{\gamma p_1/ \rho_1}) = 3(M^2-1) / [(5M^2-1)(M^2 + 3)]^{1/2}$ (10)

The wedge is formed by two walls having a small angle δ between them. After diffraction let $\overrightarrow{q_2}$,p2,p2 and S2 be the velocity vector, pressure, density and entropy at any point. For two dimensional flow we take the axes (X,Y) in the origin lying on the leading edge of wedge and X axis along the original wall produced.

If $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q}_2 . \nabla$ signifies time rate of change for a given fluid element, the equations of conservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \quad (12)$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2} \nabla p_2 = 0 \quad (13)$$

$$\frac{DS_2}{Dt} = 0 \quad (14)$$

We introduce the following transformations

$$\frac{X - q_1 t}{a_1 t} = x \quad (15)$$

$$\frac{Y}{a_1 t} = y \quad (16)$$

$$\frac{\vec{q}_2}{q_1} = (1 + u, v) \quad (17)$$

$$\frac{p_2 - p_1}{a_1 q_1 p_1} = p \qquad (18)$$

We assume that $\vec{q}_2, p_2, \rho_2$ differ by small quantities from the values (q_1,0), p_1, ρ_1 which they had before

diffraction then using equations (12), (13), (14) and (15), (16), (17) and (18) we obtain a single second order differential equation p is obtained as

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \qquad (19)$$

The equation (19) is obtained in fact using Small Perturbation Theory.

The characteristics of the differential equation given by (19) are tangents to the unit circle $x^2 + y^2=1$, so that the disturbed region is enclosed by the arc of unit circle, the diffracted shock and the wedge surface. The problem here is to obtain the pressure distribution over the diffracted shock.

The position of straight portion of the shock wave in x, y coordinates is given by

x=k where k =(ύ-q_1)/a_1 and the coordinates of the corner is (-M_1,0) where M_1=q_1/a_1

For γ =5/3, the expression for k = $((M^2+3)/(5M^2-1))^{1/2}$ …… (20)

Using Busemann Transmations and Complex Variable Technique, Lighthill (1949) has worked out a function which satisfies all the boundary conditions. From this function the pressure distribution over the diffraction shock would be obtained. The function is

$$W(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{c\,\delta\,[D(z_1 - x_o) - 1]}{(z_1^2 - 1)^{1/2}\,[\alpha - i(z_1-1)^{1/2}\,][\beta - i(z_1-1)^{1/2}\,(1/(Z_1 - x_0))]}$$

(21)

In the final z_1 plane, the imaginary part on the left hand side of (21) gives the pressure derivative which determines the pressure distribution over the diffracted shock. If one does that, then we have

$$\frac{\partial p}{\partial x_1} = \frac{c\delta}{(x_1^2 - 1)^{1/2}}\left[D - \frac{1}{(x_1 - x_o)]}\,\frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{[\alpha^2 + (x_1 - 1)][\beta^2 + (x_1 - 1)}\right. \qquad (22)$$

In (22), all the quantities are functions of the Mach number of the shock wave M except x_1, which runs from 1 to ∞ on the diffracted shock in transformed plane and is connected to y in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_2 + 1}\right)^{1/2}, k' = \sqrt{1 - k^2} \qquad (23)$$

The wall is given by y=0 so that from (23), y/k′ =0 on the wall. For this to be true x_1 should be 1 from (23). Further at the intersection of unit circle and shock wave y=k′ , so that at this point y/k′ =1. For this to be true x_1 ∞ from (23)

II. NUMERICAL SOLUTION

In order to obtain pressure distribution over the diffracted shock, the equation (22) is integrated by the method of partial fractions. The pressure p is known at $x_1=\infty$ (the point at the intersection of shock and unit circle) and the value at this point p is zero.

The pressure at y/k′ =1, the pressure at other points are obtained through integrations. The other points chosen are y/k′ =0, y/k′ =0.2, y/k′ =0.4, y/k′ =0.6, y/k′ =0.8. The equations (22) and (23) are used to get the results.

The following table gives the results after integration. The table is for y/k′ versus –p/kδ.

The Mach number of the shock wave is 1.36.

y/k’	0	0.2	0.4	0.6	0.8	1
-p/kδ	3.82	3.35	2.995	2.41	1.58	0

The table shows that –p/kδ is maximum at y/k′ =0 i.e at the point of intersection of wall and shock. From there –p/kδ falls over the diffracted and attains the value zero at at y/k′ =0 i.e at the point of intersection of shock and unit circle. From the paper of Sakurai et al (2002) it could be seen that the pressure distribution over the diffracted shock for γ =1.4 is lower than in the present case i.e for γ =5/3.

III. CONCLUSIONS

Earlier the problem was solved by Sakurai et al (2002) for γ =1.4 following Lighthill’s (1949) theory. This paper adds substantial results on pressure distributions over diffracted shock for γ =5/3. The results are quite useful for aeronautical engineers.

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